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Grand Unified Models Including Extra Z Bosons

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The grand unified theories (GUT) of the simple Lie groups including extra Z bosons are discussed. There are only SU_{5+m}, SO_{6+4n}, and E_6 under our hypothesis. First we give a general discussion for SU_{5+m}, then for SU_6 and SU_7 for illustration. We use $\underline{15}+\underline{6}^*+\underline{6}^*$ fermion representations in SU_6 but not with the fermion content, Yukawa coupling, and the hierarchy of other authors. We suggest that there is a series of "clans" of particles. These clans consist of the extra Z bosons and the corresponding fermions of the scale.

1. INTRODUCTION

Whether or not there exists a second neutral weak boson Z other than the Z of the standard $SU(2)_L \times U(1)_Y$ model is still a very interesting problem and has already been explored by a number of authors (Kang and Kim, 1976*a,b*, 1978; Barger and Phillips, 1978; Barger *et al.*, 1986; Yasve, 1978; Chanowitz *et al.*, 1979; Ross and Weiler, 1979; Zee and Kim, 1980; de Groot *et al.*, 1979; Deshpande and Iskandar, 1979*a,b*, 1980; Deshpande, 1980; Veltman, 1977; Chong-Shou Gao and Dan-di Wu, 1981). The possibility that the symmetry group is larger, such as having an extra $U(1)$ at the scale of 200 GeV and above, is not excluded by the data. Recently, Durkin and Langacker (1986) discussed the neutral current constraints on two or more Z's and obtained interesting results. Currently popular $E_8 \times E_8$ superstring models lead to an effective E_6 theory in four dimensions for the ordinary particles with at least one extra "light" Z boson. Furthermore, grand unified theories (GUT) other than SU_5 (Georgi and Glashow, 1974) generally predict the existence of extra Z bosons. This is conceivable, as they are not heavy (such as masses at the scale of 200 GeV and above). In this paper I will consider the GUT where there are extra Z bosons. There

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are no bizarre fermions in this model. The gauge group of the grand unified model is a simple group, and in the model there is a series of extra Z bosons which are not too heavy but belong to the different broken scales.

The Hypotheses

(i) Let us assume that the "observed" low-energy gauge group (Durkin and Langacker, 1986) is $SU(3)\times SU(2)\times U(1)\times U(1)\times\cdots\times U(1)$ and embedded in a simple group.

(ii) The fermion representations are complex, anomaly-free, totally antisymmetric irreducible representations (IRREPs), with no IRREP occurring more than once except for fundamental representations.

These are almost the same as Georgi's (1979) hypotheses.

A GUT that contains m, extra $U(1)$'s (to correspond with m Z bosons) as a subgroup must have rank $(4+m)$. If we consider only simple Lie groups, we have groups SU_{5+m}, SO_{9+2m}, SO_{9+2m}, SO_{8+2m}, and E_6, E_7, E_8, respectively. However, only SU_{5+4m}, SO_{6+4n} $(n=1,2,3,\ldots, m=2n-1)$, and E_6 have a complex representation (Mehta, 1966; Mehta and Srivastava, 1966).

2. THE SU_{5+m} MODEL

For SU_{5+m} the pattern of spontaneous symmetry breaking (SSB) will be adopted as follows:

$$
\begin{aligned}
SU_{5+m} &\xrightarrow{\text{adj. } H_1} SU_{5+m-1}\times U_1\\
&\xrightarrow{\text{adj. } H_2} SU_{5+m-2}\times U_1\times U_1\\
&\quad\vdots\\
&\xrightarrow{\text{adj. } H_m} SU_5\times U_1\times U_1\times\cdots\times U_1\\
&\xrightarrow{\text{adj. } H_{m+1}} SU_3\times SU_2\times \underbrace{U_1\times U_1\times\cdots\times U_1}_{m+1}\\
&\xrightarrow{\text{vect. } h_1} SU_3\times SU_2\times \underbrace{U_1\times\cdots\times U_1}_{m}
\end{aligned}
$$

$$\xrightarrow{\text{vect. } h_2} SU_3 \times SU_2 \times \underbrace{U_1 \times \cdots \times U_1}_{m-1}$$

$$\vdots$$

$$\xrightarrow{\text{vect. } h_{m-1}} SU_3 \times SU_2 \times U_1 \times U_1$$

$$\xrightarrow{\text{vect. } h_m} SU_3 \times SU_2 \times U_{1Y}$$

$$\xrightarrow{\text{vect. } h_{m+1}} SU_3 \times U_{1\text{em}} \tag{1}$$

This pattern of SSB can be realized if we use the $(m+1)$ adjoint representation of the Higgs' H_i and the $(m+1)$ vector representation of the Higgs h_i $(i=1,2,\ldots,m+1)$ and adopt the vacuum expect value (VEV) as follows (Li, 1974):

$$\begin{aligned}
\langle 0|H_1|0\rangle &= V_1 \operatorname{diag}(1,1,\ldots,1,-(4+m))\\
\langle 0|H_2|0\rangle &= V_2 \operatorname{diag}(1,1,\ldots,1,-(4+m),\ 1)\\
&\vdots\\
\langle 0|H_m|0\rangle &= V_m \operatorname{diag}(1,1,1,1,1,-(4+m),1,\ldots,1)\\
\langle 0|H_{m+1}|0\rangle &= V_{m+1} \operatorname{diag}(1,1,1,\ -\tfrac{1}{2}(3+m),\ -\tfrac{1}{2}(3+m),1,\ldots,1)\\
\langle 0|h_1^+|0\rangle &= U_1(0\,0\cdots 0\,1)\\
\langle 0|h_2^+|0\rangle &= U_2(0\ \ 0\ \cdots\ 0\ \ 1\ \ 0)\\
&\vdots\\
\langle 0|h_{m+1}^+|0\rangle &= U_{m+1}(0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \cdots\ 0)
\end{aligned} \tag{2}$$

where $V_1 > V_2 > \cdots > V_{m+1} \gg U_1 > U_2 > \cdots > U_{m+1}$ ($U_{m+1} \sim 250$ GeV). We will get a series of masses of the gauge bosons that correspond to the broken scales V_i and U_i $(i=1,2,\ldots,m+1)$, but only get the masses of the neutral gauge bosons for the broken scales U_i. These neutral gauge bosons will be called $Z_m, Z_{m-1},\ldots,Z_2, Z_1, Z$ and their masses will not be too heavy,

$$M_{z_m} > M_{z_{m-1}} > \cdots > M_{z_2} > M_{z_1} > M_z$$

We shall define the value of the charge of m $U(1)$ for (1) in order to make the $(6+m-\mathrm{i})$th component of the h_i a neutral color singlet which will be developed into the VEV. For the SSB of the vectors in (1) we may also unite the vector and adjoint Higgs

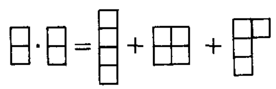
$$\langle 0|H_i'|0\rangle - \langle 0|h_i|0\rangle \tag{3}$$

in order to obtain a more neat and tidy SSB. The gauge bosons are

$$A = \frac{1}{\sqrt{2}} \boldsymbol{\lambda} \cdot \mathbf{A} \tag{4}$$

where λ^i are the (n^2-1) generators of SU_n. Fermion IRREPs are an antisymmetric product of two fundamentals, $\psi_L^{ab} = -\psi_L^{ba}$ and the $(m+1)$ $(5+m)^*$-dimensional ψ_{aL}. The sum of these fermion IRREPs is anomaly free. The fermions in the antisymmetrical ψ_L^{ab} after the fifth row for $m \geq 1$ are all heavy, in contrast with present energies. When the m increase, these heavy fermions will be heavier and heavier. Because of the left-hand fermions after the fifth row in ψ_L^{ab} and the corresponding left-hand antifermions in m $\psi_{aL}^{\gamma+1}$ $(\gamma = 1, 2, \ldots, m)$ can form Yukawa coupling and their scales of VEV are large.

The decompositions of direct products of fermion representation are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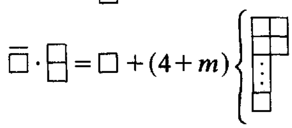
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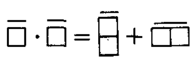
(7)

So the adjoint representation of the Higgs does not couple to fermions. This is very fortunate because otherwise the natural scale for fermion masses would be the grand unified scale. However, $(m+1)$ vector Higgs h_i can form the Yukawa coupling from (6),

$$g\psi_{aL}^{iT} C \psi_L^{ab} h_{bj}^{+} + \text{H.C.}, \qquad i, j = 1, 2, 3, \ldots, (m+1) \tag{8}$$

If we use the VEV of the $\langle 0|h_i^{+}|0\rangle$ in (2), then we will get the masses of the fermions

$$M_d = M_e, \qquad M_D = M_E, \qquad M_{D'} = M_{E'}, \ldots, M_{D^{m-1}} = M_{E^{m-1}} \tag{9}$$

These fermions are contained in ψ_L^{ab} and ψ_{aL}. We will discuss equation (9) later. These fermions and the corresponding Z_m boson of the scales will be called "clans." The Z_m will be a clannhead. If we introduce the Higgs H^{ab} of the antisymmetric product of two fundamentals, then it can form the Yukawa coupling

$$g' \varepsilon_{abcdef} (\psi_L^{ab})^T C \psi_L^{cd} H^{ef} + \text{H.C.} \tag{10}$$

If we use

$$\langle 0|H^{ab}|0\rangle = \nu(\delta^{a5}\delta^{b6} - \delta^{a6}\delta^{b5}) \tag{11}$$

then we may get the u quark mass, which is the same as that of SU_5, but equations (10) and (11) do not generate the masses of the gauge bosons, where $\nu \sim U_{m+1}$.

For the $\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$ of equation (7) there is yet Yukawa coupling

$$\psi_{aL}^{T} C \psi_{bL} H^{ab} + \mathrm{H.C.} \tag{12}$$

but (11) and (12) do not generate the masses of the fermions. In order to illustrate SU_{5+m}, let us look at SU_6 and SU_7.

If the GUT is SU_6, then there is only one extra Z boson. This will be a very interesting case (Durkin and Langacker, 1986). A GUT-based on SU_6 has already been explored by a number of authors (Baaklini, 1980; Langacker *et al.*, 1978*a,b*; Abud *et al.*, 1977; Yosbimura, 1977; Frampton and Nandi, 1979; Frampton, 1979, 1980; Inoue *et al.*, 1977*a,b*; Yun, 1978, 1979; Kim and Roiesnel, 1980; Chakrabarti *et al.*, 1980) in directions different from ours. Most of their models are vectorlike and contain electroweak $SU(3)\times U(1)$. These are now ruled out by neutral current data. In this paper the SU_6 model is different from theirs. Because the GWS (Weinberg, 1967) model is in excellent agreement with existing data, our SU_6 model will keep all results of the GWS model under the scale of the present energy. The extra Z boson and the corresponding fermions of the scale, we call it a clan, will appear under the scale of the high energy. The "generations" are included in a clan; one clan includes many generations. The difference of the mass scales between generations is small (for example, between the order of MeV and 10^2 GeV), while the difference of the scale between clans is large (between the order of 10^2 GeV and 10 TeV for SU_6). All quarks and leptons in the same generation will be called a "family," so a clan is also different from a family. In this paper the model is independent of generation but depends on the clan.

2.1. The Case of SU_6 (One Extra Z Boson)

For SU_6 the pattern (1) of the SSB will be

$$\begin{aligned} SU(6) &\xrightarrow{\text{adj. } H_1,\, M_6} SU(5)\times U(1)_a \\ &\xrightarrow{\text{adj. } H_2,\, M_5} SU(3)_c\times SU(2)\times U(1)_b\times U(1)_a \\ &\xrightarrow{\text{vect. } h_1,\, M_1} SU(3)_c\times SU(2)\times U(1)_Y \\ &\xrightarrow{\text{vect. } h_2,\, M_w} SU(3)_c\times U(1)_{\mathrm{em}} \end{aligned} \tag{1'}$$

and the expanded formula of the gauge bosons (4) may be written

(Langacker, 1980)

$$(A_\mu)=\begin{pmatrix} G_1^1-\frac{2}{\sqrt{30}}B+\frac{A'}{\sqrt{30}} & G_2^1 & G_3^1 & \bar{X}^1 & \bar{Y}^1 & \bar{S}^1 \\ G_1^2 & G_2^2-\frac{2}{\sqrt{30}}B+\frac{A'}{\sqrt{30}} & G_3^2 & \bar{X}^2 & \bar{Y}^2 & \bar{S}^2 \\ G_1^3 & G_2^3 & G_3^3-\frac{2}{\sqrt{30}}B+\frac{A'}{\sqrt{30}} & \bar{X}^3 & \bar{Y}^3 & \bar{S}^3 \\ X_1 & X_2 & X_3 & \frac{W^3}{\sqrt{3}}+\frac{3}{\sqrt{30}}B+\frac{A'}{\sqrt{30}} & W^+ & \bar{S}^4 \\ Y_1 & Y_2 & Y_3 & W^- & \frac{-W^3}{\sqrt{3}}+\frac{3}{\sqrt{30}}B+\frac{A'}{\sqrt{30}} & \bar{S}^5 \\ S_1 & S_2 & S_3 & S_4 & S_5 & \frac{-5}{\sqrt{30}}A' \end{pmatrix} \tag{4'}$$

where we will define

$$Z_1\sim\frac{-5}{\sqrt{30}}A', \qquad Z\sim\frac{-W^3}{\sqrt{3}}+\frac{3B}{\sqrt{30}}+\frac{A'}{\sqrt{30}}, \qquad A\sim\frac{W^3}{\sqrt{3}}+\frac{3B}{\sqrt{30}}+\frac{A'}{\sqrt{30}}$$

The (2) of the Higgs and their VEV will be

$$\begin{aligned} \langle 0|H_1|0\rangle &= V_1\,\mathrm{diag}(1,1,1,1,1,-5) \\ \langle 0|H_2|0\rangle &= V_2\,\mathrm{diag}(1,1,1,-2,-2,1) \\ \langle 0|h_1^+|0\rangle &= U_1(0\ 0\ 0\ 0\ 0\ 1) \\ \langle 0|h_2^+|0\rangle &= U_2(0\ 0\ 0\ 0\ 1\ 0) \end{aligned} \tag{2'}$$

The SSB of (1′) may be realized by (2′) and we obtain the masses of the gauge bosons in proper order,

$$
\begin{aligned}
M_S^2 &\sim 72 V_1^2 \\
M^2_{X,\, Y,\, (S_{4,5}-S_{1,2,3})} &\sim 18 V_2^2 \\
M^2_{Z_1,\, S} &\sim U_1^2 \\
M^2_{w,\, z,\, (Y-Z)} &\sim U_2^2
\end{aligned}
\tag{13}
$$

If we use equation (3), that is,

$$
(\langle 0|H_1'|0\rangle - \langle 0|h_1|0\rangle) \tag{3'}
$$

for SU_6, then $SU(3)\times SU(2)\times U(1)\times U(1)$ may also be broken to $SU(3)\times SU(2)\times U(1)$ and we get only the mass of the Z_1,

$$
M^2_{Z_1} \sim U_1^2
$$

where the H_1' is third adjoint representation of the Higgs and its VEV adopts

$$
\langle 0|H_1'|0\rangle = \frac{U_1}{6\sqrt{2}}\,\mathrm{diag}(1, 1, 1, 1, 1, -5) \tag{14}
$$

Fermion masses. The left-handed fermions are assigned to one 15 and both 6*, 6*-dimensional representations for SU_6; their manifest forms are

$$
(\psi^{ab})_L = \frac{1}{\sqrt{2}}\begin{pmatrix}
0 & U_3^c & -U_2^c & -U^1 & -d^1 & -D^1 \\
-U_3^c & 0 & U_1^c & -U^2 & -d^2 & -D^2 \\
U_2^c & -U_1^c & 0 & -U^3 & -d^3 & -D^3 \\
U^1 & U^2 & U^3 & 0 & -e^+ & -E^+ \\
d^1 & d^2 & d^3 & e^+ & 0 & -E^{0c} \\
D^1 & D^2 & D^3 & E^+ & E^{0c} & 0
\end{pmatrix}_L \tag{15}
$$

$$
(\psi_a')_L = \begin{pmatrix}
d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \\ \nu_{E^0}
\end{pmatrix}_L \tag{16}
$$

$$(\psi_b^2)_L = \begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \\ E^- \\ -E^0 \\ \nu_{E^-} \end{pmatrix}_L \tag{17}$$

where the $\underline{10}$ in $\underline{15}$ and the $\underline{5}^*$ in $\underline{6}^*$ are just $\underline{10}+\underline{5}^*$ in SU_5. It will keep all results of the standard model. The $\underline{5}$ in $\underline{15}$ and the $\underline{5}^*$ in another 6^* may form heavy fermions. These heavy fermions and extra Z (that is Z_1) form a clan. The energy is enough to enable the discovery of the particles of this clan on an SSC accelerator.

We may obtain the masses of the fermions from equations (8) and (10)-(11) of the Yukawa coupling,

$$M_d = M_e = U_2 g \tag{18}$$

$$M_D = M_E = U_1 g \tag{19}$$

$$M_u = 8\nu g' \tag{20}$$

these results are similar to SU_5 (Georgi *et al.*, 1974; Buras *et al.*, 1978; Nanopoulos and Ross, 1979). It is observed easily from equations (13) and (19) that the D and Z_1 have been broken at same scale U_1, so they will form a clan.

An interesting case is that the $\underline{15}$, both $\underline{6}^*$ and $\underline{6}^*$ in SU_6 may be $\underline{27}$ in E_6, which is a currently popular effective E_6 superstring model.

A direct asymptotic calculation of $\sin^2\theta_w$ yields

$$\sin^2\theta_w = \frac{\mathrm{Tr}\, I_3^2}{\mathrm{Tr}\, Q^2} \tag{21}$$

where $q^2 \geq M_6^2$. Equation (21) is independent of the scales of the intermediate breaking in equation (1′) and depends only on the charge operator. It is also correct for more $U(1)$ such as above (1). Its form is the same as SU_5. If we select

$$Q = \mathrm{diag}(-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, 1, 0, 0) \tag{22}$$

then $\sin^2\theta_w = \frac{2}{8}$. The q^2 dependence of $\sin^2\theta_w$ can be calculated from the renormalization group equations (Georgi *et al.*, 1974; Buras *et al.*, 1978; Nanopoulos and Ross, 1979), but it contains more parameters, such as more thresholds in the region between M_w and unified scale M_6, mixing angle between $U(1)_a$ and $U(1)_b$, more β, etc. So reasonable values of $\sin^2\theta_w$ and proton decay can be accommodated.

2.2. The Case of $SU(7)$ (Two Extra Z Bosons)

For $SU(7)$ the pattern (1) of SSB will be

$$
\begin{aligned}
SU(7) &\xrightarrow{\text{adj. } H_1} SU(6)\times U(1)\\
&\xrightarrow{\text{adj. } H_2} SU(5)\times U(1)\times U(1)\\
&\xrightarrow{\text{adj. } H_3} SU(3)_c\times SU(2)\times U(1)\times U(1)\times U(1)\\
&\xrightarrow{\text{vect. } h_i} SU(3)_c\times SU(2)\times U(1)\times U(1)\\
&\xrightarrow{\text{vect. } h_2} SU(3)_c\times SU(2)\times U(1)_Y\\
&\xrightarrow{\text{vect. } h_3} SU(3)_c\times U(1)_{\text{em}}
\end{aligned}
\tag{1''}
$$

The expanded formula of the gauge bosons (4) is similar to (4′) except for an increase of 13 gauge bosons. The Higgs and their VEV of the (2) will be

$$
\begin{aligned}
\langle 0|H_1|0\rangle &= V_1\,\text{diag}(1, 1, 1, 1, 1, 1, -6)\\
\langle 0|H_2|0\rangle &= V_2\,\text{diag}(1, 1, 1, 1, 1, -6, 1)\\
\langle 0|H_3|0\rangle &= V_3\,\text{diag}(1, 1, 1, -\tfrac{5}{2}, -\tfrac{5}{2}, 1, 1)\\
\langle 0|h_1^+|0\rangle &= U_1(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1)\\
\langle 0|h_2^+|0\rangle &= U_2(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0)\\
\langle 0|h_3^+|0\rangle &= U_3(0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0)
\end{aligned}
\tag{2''}
$$

The U_1 is a scale of the mass of the extra Z_2. The Z_2 and corresponding fermions of the scale will form a clan. The U_2 is the scale of Z_1. The Z_1 and corresponding fermions of the scale will form another clan.

The fermion IRREPs are 21 and three 7^*-dimensional for $SU(7)$,

$$
(\psi^{ab})_L = \begin{pmatrix}
0 & U_3^c & -U_2^c & -U^1 & -d^1 & -D^1 & -D'^1\\
 & 0 & U_1^c & -U^2 & -d^2 & -D^2 & -D'^2\\
 & & 0 & -U^3 & -d^3 & -D^3 & -D'^3\\
 & & & 0 & -e^+ & -E^+ & -E'^+\\
 & & & & 0 & -E^{0c} & -E'^{0c}\\
 & & & & & 0 & -l^c\\
 & & & & & & 0
\end{pmatrix}_L
\tag{23}
$$

$$(\psi^1)_L = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \\ \nu_{E^0} \\ \nu_l \end{pmatrix}_L \tag{24}$$

$$(\psi^2)_L = \begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \\ E^- \\ -E^0 \\ \nu_{E^-} \\ \nu_{E'^0} \end{pmatrix}_L \tag{25}$$

$$(\psi^3)_L = \begin{pmatrix} D_1'^c \\ D_2'^c \\ D_3'^c \\ E'^- \\ -E'^0 \\ l \\ \nu_{E'^-} \end{pmatrix}_L \tag{26}$$

where the $\underline{15}$ in $\underline{21}$ and both $\underline{6^*}$ and $\underline{6^*}$ in both $\underline{7^*}$ and $\underline{7^*}$, are just $\underline{15}+\underline{6^*}+\underline{6^*}$ in SU_6. The $\underline{6}$ in $\underline{21}$ and $\underline{6^*}$ in the third $\underline{7^*}$ may form heavier fermions. These heavier fermions and Z_2 form another clan.

We may get the fermion masses

$$M_d = M_e, \qquad M_D = M_E, \qquad M_{D'} = M_{E'} \tag{27}$$

and the U quark mass from equations (8), (10), and (11). The M_D, M_E, and Z_2 make up another clan.

We know that the case of SU_{5+m} is similar to $SU(7)$ and $SU(6)$ from above. So equation (9) of SU_{5+m} may be obtained from the extension of equation (27) and equations (18), (19).

3. THE SO_{6+4n} MODEL

The SO_{6+4n} are all real representations under SO_{10} for $n \geq 2$ [$n = (m+1)/2$]. It is not very interesting.

The SO_{10} $(n=1, m=1)$ has been discussed by many authors (e.g., Rajpoot, 1980; Georgi and Nanopulos, 1979*a*–*c*; Fritzsch and Minkowski, 1975; Chanowitz *et al.*, 1977):

$$SO_{10} \xrightarrow[M_G]{\text{adj.}} SU_5 \times U_1 \xrightarrow[M]{\text{adj.}} SU_3 \times SU_2 \times U_1 \times U_1$$

If $M_G \gg M$, then the predictions of $\sin^2\theta_w$, etc., are the same as SU_5.

4. THE E_6 MODEL

The E_6 model has been discussed by many authors. Some of the models contain the $SU_3^c \times SU_3 \times SU_3$ subgroups for which there is no t quark and the τ^- neutral currents are pure vector. So these models are not very attractive. Barbieri and Nanopulos (1980), Ramond (1979), and Gell-Mann *et al.* (1979) have discussed an interesting pattern of SSB,

$$E_6 \to SO_{10} \times U_1 \to SU_5 \times U_1 \times U_1$$

They argue that the 10 multiplet fermions in $\underline{27}$ will be heavy and the other $\underline{15}$ fermions in $\underline{27}$ have small masses. Another interesting pattern is

$$E_6 \to SU_6 \times SU_2$$

where $\underline{27}$ multiplet fermions may be decomposed into $(15, 1)+(6^*, 2)$; if the doublet in $SU(2)$ is decomposed into 6_u^* and 6_d^*, then they may correspond to both 6^* and 6^* in SU_6. So the SU_6 may also be embedded in E_6.

5. SUMMARY

Because the GWS $SU(2) \times U(1)$ model has been extended to $SU(2) \times U(1) \times U(1) \times \cdots \times U(1)$, we would like to embed $SU(3) \times SU(2) \times U(1) \times \cdots \times U(1)$ in a simple group. All simple Lie groups of the contribution have only SU_{5+m}, SO_{6+4n}, and E_6 under our hypothesis.

For SU_{5+m} the pattern of the SSB are shown in equation (1) and Figure 1; these may be realized by $(m+1)$ adjoint and $(m+1)$ vector representations of the Higgs. Fermions are assigned to one antisymmetric product of two fundamentals and $(m+1)$ Hermitian conjugate representations of the $(5+m)$ dimension. Fermion masses cannot be superheavy because the adjoint representation of the Higgs cannot couple to fermions. The down-quark and lepton masses are contributed by $(m+1)$ vector Higgs, the neutrinos are massless, and up-quark masses are contributed by an antisymmetric Higgs. There are no up quarks, only down quarks, heavy leptons, and their neutrinos, except at the scale of the present energy, where the

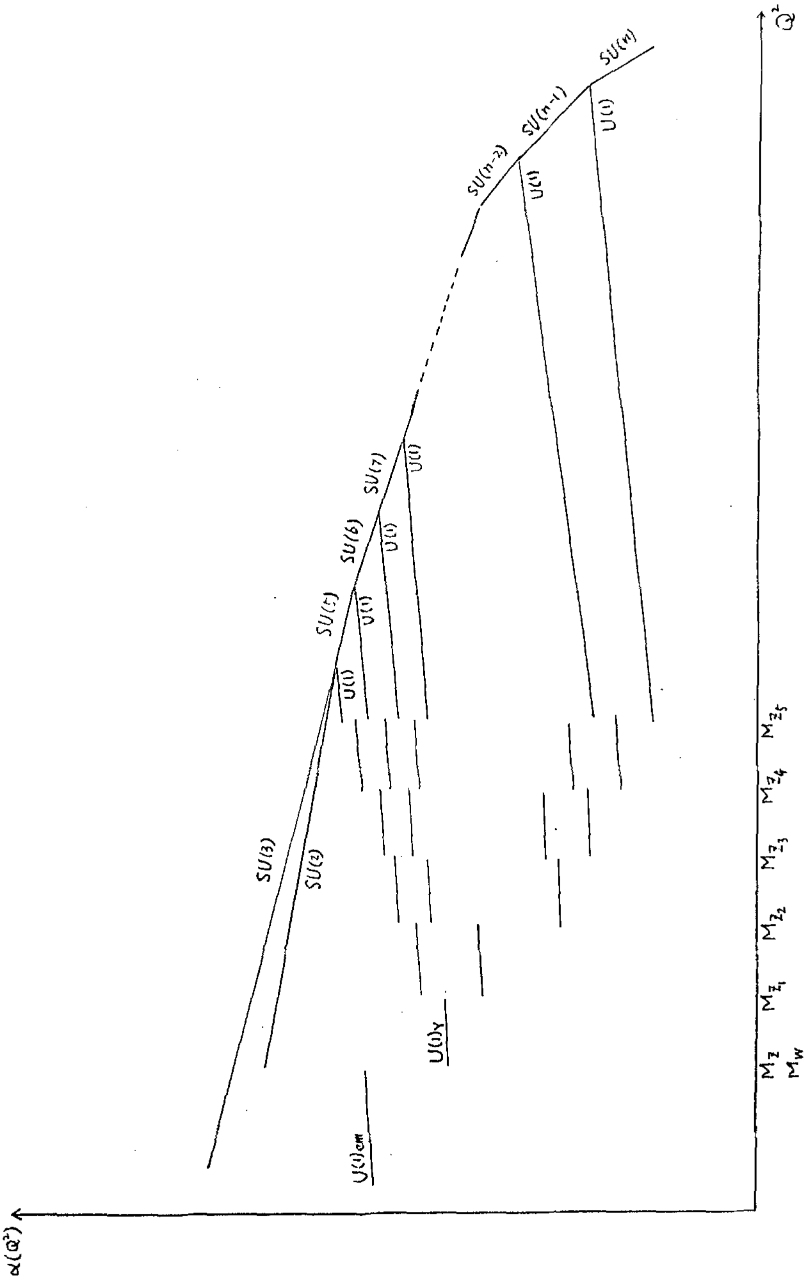

Fig. 1. The pattern of SSB of SU_{5+m}.

fermions and gauge bosons are the same as in the GWS model. They and corresponding extra Z bosons form self-clan respectively. Finally, I emphasize what seems to be the main feature of the class of models discussed here, that is, the existence of more stages of SSB characterized by radically different mass scales. These may be referred to as the blooming of the desert and many parameters may be accommodated in order to get the expected results, such as proton decay, $\sin^2\theta_w$, and so on. However, in the context of the conventional Higgs mechanism, no natural explanation can be found of the ratio of heavy mass to ordinary mass. This leads us to consider the gauge hierarchy problem from a different viewpoint.

The SO_{6+4n} and E_6 have been discussed by a number of authors.

Because the quarks on the scale of high energy are all down quarks, the spectrum of the hadrons which are composed of these down quarks is different from hadrons at present.

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